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Preliminary design and optimization of slotted tube grain for solid rocket motor

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Abstract : In this paper , design and optimization technique of slotted tube grain for solid rocket motors has been discussed. In doing so , the design objectives and constraints have been set , geometric parameters identified , performance prediction parameters calculated , thereafter preliminary designs completed and finally optimal design reached.

Geometric model for slotted tube grain configuration has been developed. Average thrust has been taken as the objective function with constraints of burning time , mass of propellant , fixed length and diameter of chamber case. Lumped parameter method has been used for calculating the performance prediction parameters. A set of preliminary designs has been completed and an analysis of these results conducted. Although all the preliminary results fulfill the design requirements in terms of objective function and constraints , however in order to attain the optimal design , Sequential quadratic programming optimization technique has been adopted. As the slotted tube grain geometry is totally dependent upon various independent variables and each of these variables has a bearing on explicit characteristic of grain designing , hence affects of the independent variables on performance parameters have been examined , thus variation laws have been developed. Basing on the variation laws and the analysis of preliminary design results , upper and lower limits have been defined for the independent geometric variables and an initial guess provided for conducting optimization. Results attained exhibits that an optimal result has been attained and the value of objective function has been maximized. All the design constraint limits have also been met while ensuring sound values of volumetric loading fraction , web fraction and neutrality.

This methodology of design and optimization of slotted tube grain for solid rocket motors can be used by engineers as a reference guide for actual design and engineering purposes.

Key words : solid rocket motor ; slotted tube grain ; internal ballistics ; optimization

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Nomenclature

a burn rate coefficient
 A_b burning area
 A_{bi} initial burning area
 A_e nozzle exit area
 A_p port area
 A_{pc} port area of cylindrical portion
 A_{ps} port area of slotted portion
 A_t throat area
 BR burning rate

c^* characteristic velocity
 C_F thrust coefficient
 d_e nozzle exit diameter
 d_t throat diameter
 e burning web
 F thrust
 F_{av} average thrust
 F_{max} maximum thrust
 I_s specific impulse
 I_t total impulse

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L_c	length of cylinder
L_s	length of slot
m_p	mass of propellant
n_{eu}	neutrality
N	number of slot points
n	pressure sensitivity coefficient
p_{av}	average chamber pressure
p_c	chamber pressure
P_b	burning perimeter
P_{bc}	burning perimeter of cylindrical portion
P_{bs}	burning perimeter of slotted portion
p_{max}	maximum chamber pressure
R	grain outer radius
r	grain inner radius
t_b	burning time
V_l	volumetric loading fraction
V_m	motor volume
V_p	grain volume
W	web thickness
w_f	web fraction
x	half width of slot
ζ	angular fraction
η	slot valley angle

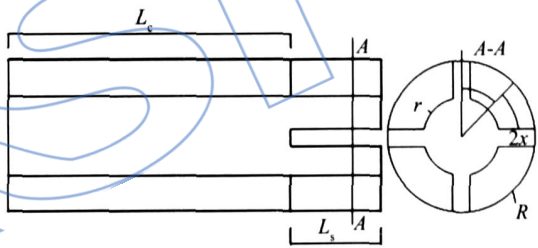
Introduction

Grain design is essentially a geometrical constraint depending upon the thrust-time and pressure-time laws^[1]. These laws define the solid rocket motor (SRM) performance that in turn affects the ballistic performance of any missile and launch vehicle.

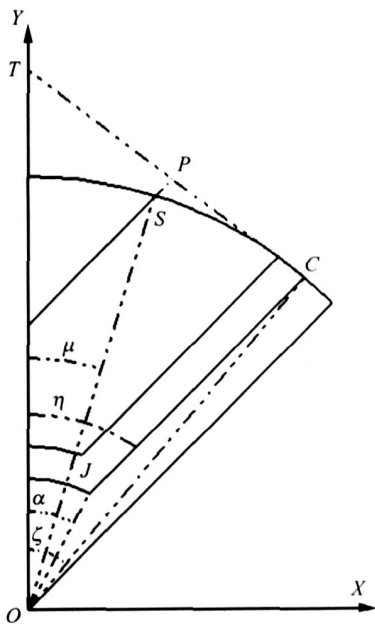
The scope of this study is to develop an efficient slotted tube grain (STG) design approach for SRMs. The STG configuration consists of a cylindrical tube of propellant into which a number of slots have been cast. In tubular grain, to make burning area A_b constant, the grain must be slotted^[2]. The A_b decreases in the slotted portion so the shape and length of slot can be altered to satisfy design requirements. The configuration offers some significant advantages for the designers. Especially volumetric loading fraction V_l up to 95 % are possible to be attained by using these configurations, however the major design constraints like mass of propellant m_p play a key role in deciding its value. V_l ranging between 46 to 95.5 % has been used in motors ranging from

50 to 580 mm in diameter. The designs have lived up to expectations^[3].

The main outputs of the ballistic prediction are the chamber pressure and thrust^[4]. In this paper lumped parameter method will be used for performance prediction. Calculations will be performed to obtain performance prediction parameters basing on the design objectives and constraint limits. The preliminary designing will be completed and thereafter affects of all the independent design variables on slotted tube performance will be examined and analyzed. Finally optimization of the same will be conducted by keeping average thrust F_{av} as the objective function. The burning time t_b , propellant mass m_p , fixed length L and radius R of grain will be taken as major constraints. Thus an optimal design will be reached by using this methodology.



(a) Basic parameters



(b) Burning zones

Fig. 1 Geometric definition of STG

1 Geometric model of STG

In case of tube or cylindrical grains, for adjusting large values of burning area, long lengths of the grain may have to be incorporated that may not be feasible, hence slots can be put across the grain or in cross sections^[5]. The shapes and number of slots can be adjusted according to the requirements. Slotted tube basic parameters, geometric definition and burning zones have been defined in Fig. 1.

The burning surface of a STG may pass through different phases during the course of its consumption. Initially the propellant will burn outwards in the interior cylinder, while sideways and lengthwise in the slots. Eventually the slots come close enough together and the curved cylindrical interior surface separating adjacent slots will disappear into a line contact between the two slots. As the burning continues further, the propellant between the adjacent slots diminishes rapidly and will vanish thus leaving behind only

$$\eta = \pi/N, \quad \zeta = \frac{\pi}{N} - \sin^{-1} \frac{x}{R}, \quad OC = R, \quad TC = OC \sin \zeta \cos \eta, \quad Y_t = OT = OC(\cos \zeta + \sin \zeta \tan \eta)$$

$$\alpha = \sin^{-1} \left(\frac{X_i}{r+e} \right), \quad X_i = \frac{-K \cot \eta + \sqrt{(K \cot \eta)^2 - (1 + \cot^2 \eta)[K^2 - (r-e)^2]}}{1 + \cot^2 \eta}$$

$$\mu = \sin^{-1} \frac{X_s}{R}, \quad K = Y_t - \frac{TC-e}{\sin \eta}$$

$$X_s = \frac{-n(Y_p - nX_p) + \sqrt{n^2(Y_p - nX_p)^2 - (1+n^2)[(Y_p - nX_p)^2 - R^2]}}{1+n^2}$$

$$n = \tan \left(\frac{\pi}{2} - \eta \right), \quad X_p = (TC - e) \cos \eta, \quad Y_p = Y_t - (TC - e) \sin \eta$$

Equations to determine burning perimeter P_{bs} and port area A_{ps} for slotted portion, having two zones, are as follows:

Zone 1

$$P_{bs} = \frac{X_s}{\sin \eta} - \frac{X_i}{\sin \eta} + (r+e)\alpha$$

$$A_{ps} = \frac{R^2}{2} \left(\frac{\pi}{N} - \mu \right) + \frac{R}{2} (r^2 + e) \sin(\mu - \alpha) + \frac{(r+e)^2}{2} \alpha$$

Zone 2

$$P_{bs} = \frac{X_s}{\sin \eta}$$

$$A_{ps} = \frac{R^2}{2} \left(\frac{\pi}{N} - \mu \right) + \frac{R}{2} \left(Y_t - \frac{TC-e}{\sin \eta} \right) \sin \mu$$

The cylindrical section has only one zone of

unslotted cylindrical section of the grain. During the course of burning the cylindrical section burns progressively whereas the slotted region burns regressively thus achieving neutrality. In the current study the front and aft ends of the grain have been considered to be inhibited for burning.

The burning zones for slotted portion can be defined as:

$$0 \leq e \leq \frac{r \sin \eta - x}{1 - \sin \eta} \quad \text{Zone 1}$$

$$\frac{r \sin \eta - x}{1 - \sin \eta} < e \leq R \sin \eta - x \quad \text{Zone 2}$$

The slotted tube configuration is defined by N, x, r, L_s, L_c, R . Calculations of the geometric evolution have been conducted using basic geometry and trigonometric relations. Various relationships have been derived with respect to each zone in order to find the burning perimeter and thereafter A_b for the zones. Equations to determine geometric relation for each zone are as follows^[6]:

burning and the web limit for this section is:

$$0 \leq e \leq R \sin \eta - x$$

Equations to determine geometric relation for this zone are as follows:

$$P_{bc} = 2\pi(r+e), \quad A_{pc} = \pi(r+e)^2$$

Finally the total burning surface area has been calculated by considering following two phases:

Phase 1 When cylindrical and slot portions are burning.

$$0 \leq e \leq R \sin \eta - x$$

$$A_b = (L_s + e) P_{bs} + (L_c - e) P_{bc} + A_{ps}$$

Phase 2 When slot portion has finished burning and only cylindrical portion is left.

$$R \sin \eta - x < e \leq R$$

$$A_b = (L_c - e) P_{bc} + \pi R^2 - \pi (r + e)^2$$

Web fraction w_f is the ratio of web thickness to outer radius of grain , i. e.

$$w_f = \frac{R - r}{R}$$

Volumetric loading fraction V_l is ratio of propellant volume to the chamber volume , i. e.

$$V_l = \frac{V_m}{V_p}$$

2 Performance prediction

The performance prediction of SRMs has obvious importance in their designing. Definition of the ballistic performance parameters is related to thrust-time and pressure-time curves for the motor. The main outputs of the performance predictions are the p_{av} , p_{max} , F_{av} , F_{max} and t_b . Other important parameters that influence the performance have also been calculated in this study.

Lumped parameter method has been used for performance prediction in which the combustion products have been assumed to be ideal gases and chamber conditions uniform i. e. chamber pressure and chamber temperature remain constant. The chamber pressure is calculated as:

$$p_c = \left(\rho_p a c^* \frac{A_b}{A_t} \right)^{\frac{1}{1-\eta}}$$

The pressure is used for calculation of thrust :

$$F = C_F p_c A_t$$

The neutrality refers to flatness of the thrust-time curve. Lesser values of neutrality allow having higher m_p and F_{av} values. The neutrality is defined as :

$$n_{eu} = \frac{F_{max}}{F_{av}}$$

3 Preliminary design and optimization

3.1 Preliminary design

3.1.1 Design objective

- 1) Grain length : 3700 mm
- 2) Mass of propellant : (350 ± 10) kg
- 3) Burning time : (4.5 ± 1) s.

4) Average thrust : ≥ 240 kN

5) Grain outer radius : 147 mm

3.1.2 Propellant parameters

Considering the mechanical properties and internal ballistics required , the following propellant parameters have been selected :

- 1) Propellant : HTPB/ AP/ AL
- 2) Density : 1.74 g/ cm³
- 3) c^* : 1554 m/ s

3.1.3 Nozzle parameters

- 1) Exit diameter : ≤ 280 mm
- 2) A_e/ A_t : 7.6

3.1.4 Preliminary design results

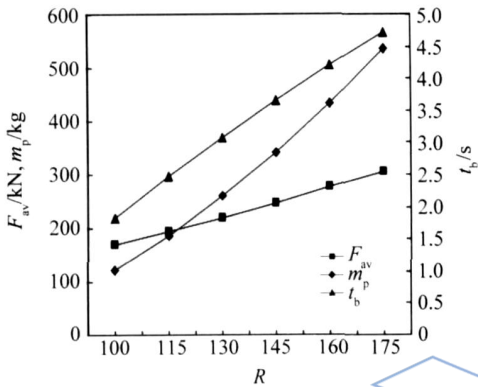
From the design objectives it can be observed that the required length to diameter (L/ D) ratio is very high , i. e. > 12 , due to this reason very high operating pressures are expected during operation of SRM. Due to high L/ D ratio , the burning area under the domes available for propellant is relatively very small as compared to total burning area available for grain to be cast hence the affects due to domes on the internal ballistic parameters will be negligible. STG configuration has a wide range of geometries that can be used , a network of designs was generated. Results of some of these preliminary design indices are placed in Table 1.

Table 1 Preliminary results of STG

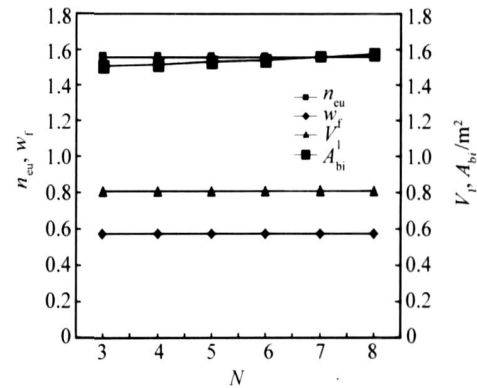
	Preliminary design 1	Preliminary design 2	Preliminary design 3
x / mm	10	15	12
r / mm	62	63	63
L_s / mm	410	200	92.5
L_c / mm	3290	3500	3607.5
N	4	4	4
F_{av} / kN	240	243	243.2
m_p / kg	354	357	360
t_b / s	3.96	3.92	3.91
p_{av} / MPa	32.4	32.9	33.3
n_{eu}	1.4	1.5	1.53
w_f	0.578	0.57	0.57
V_l	0.81	0.83	0.84

Preliminary design 3 is giving the best re-
sults amongst the above mentioned preliminary
designs, hence it has been selected to conduct
the sensitivity analysis. Fig. 2 shows the varia-
tion in various parameters caused by R , N , r ,
 L_s , L_c , χ on STG performance.

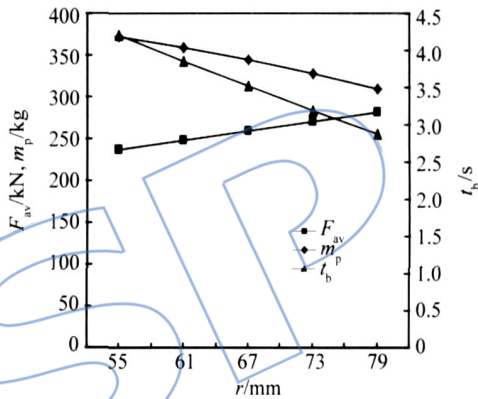
The independent geometric parameters of
STG affect the IB performance. The affects of
these parameters on grain design have been ana-
lyzed analytically so as to ascertain their influ-
ence on design parameters.



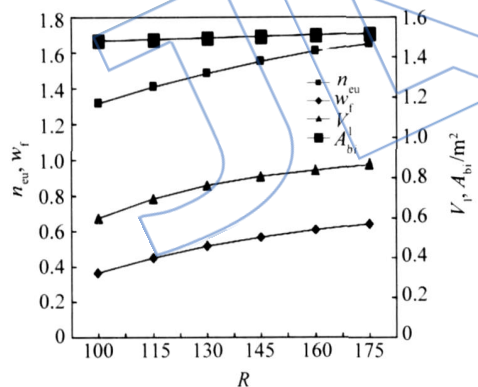
(a) F_{av}, m_p, t_b vs R



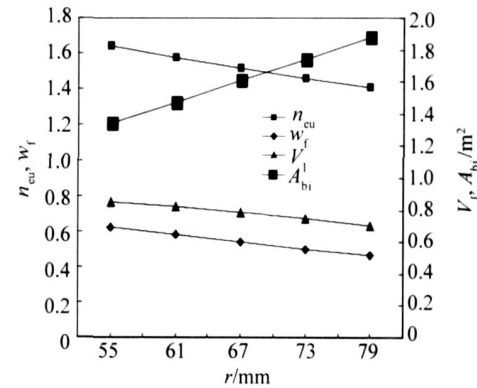
(d) n_{cu}, w_f, V_l, A_{bi} vs N



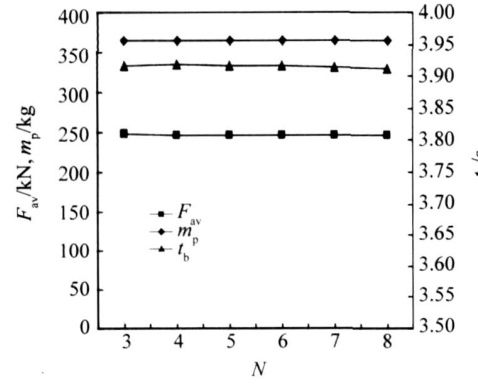
(e) F_{av}, m_p, t_b vs r



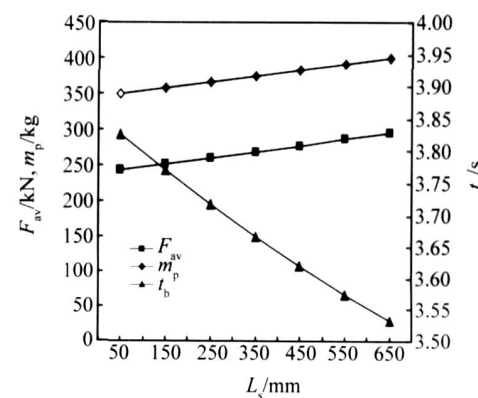
(b) n_{cu}, w_f, V_l, A_{bi} vs R



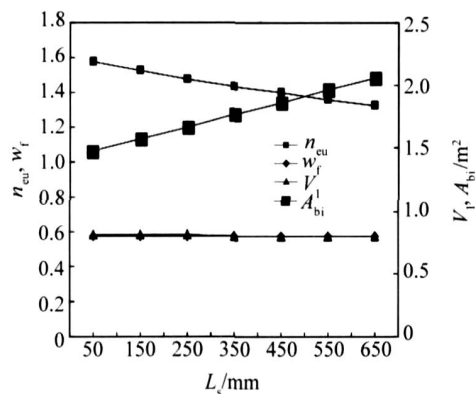
(f) n_{cu}, w_f, V_l, A_{bi} vs r



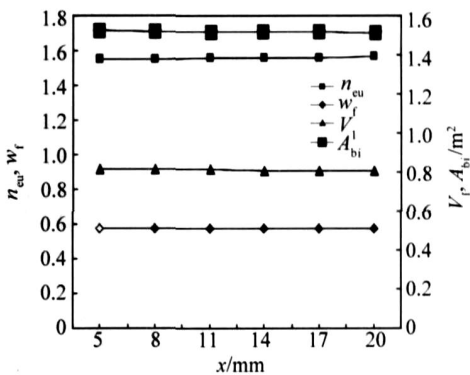
(c) F_{av}, m_p, t_b vs N



(g) F_{av}, m_p, t_b vs L_s

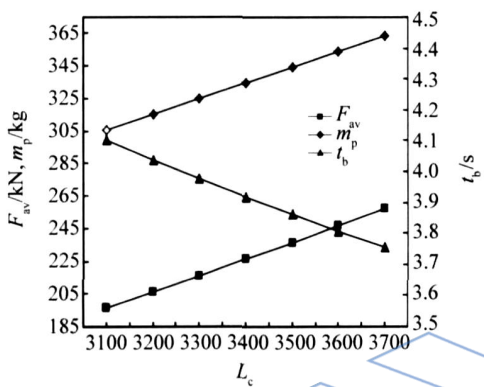


(h) n_{cu}, w_f, V_l, A_{bi} vs L_s

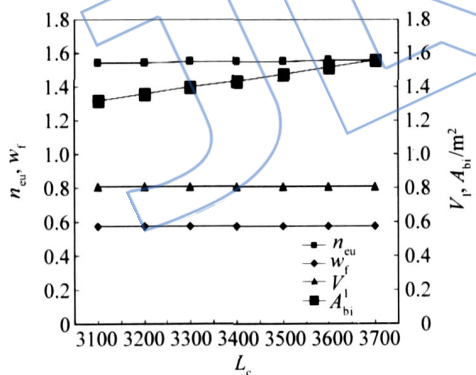


(i) n_{cu}, w_f, V_l, A_{bi} vs x

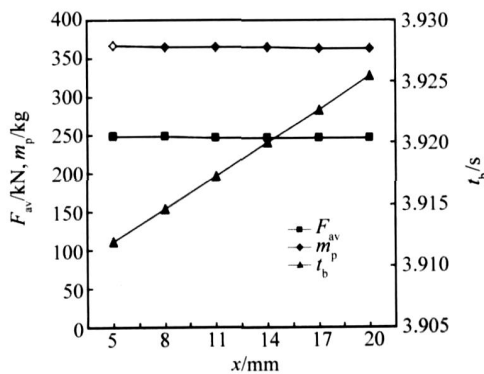
Fig.2 Variations caused by R, N, r, L_s, L_c, x



(j) F_{av}, m_p, t_b vs L_c



(k) n_{cu}, w_f, V_l, A_{bi} vs L_c



(l) F_{av}, m_p, t_b vs x

3.1.5 Affects of angle coefficient R

As shown in Fig. 2(a) and Fig. 2(b), the defined range for R was $100\text{ mm} \leq R \leq 175\text{ mm}$. By increasing R , the values of m_p , F_{av} and t_b increases. V_l , w_f and A_{bi} also increase, but A_{bi} increases slightly because of short length of the slotted portion. Better neutrality is attained as value of R is decreased.

3.1.6 Affects of N

From Fig. 2(c) and Fig. 2(d), by increasing N , the values of F_{av} , m_p , t_b and V_l decrease and A_{bi} increases. w_f remains constant. It can be observed that in the preliminary design 3, the value of L_s is 92.5 mm which is very small as compared to L_c . Hence variations in the other parameters caused by changing number of slots in slotted portion are less pronounced in this case. These variations will be much more pronounced in designs where L_s will be much higher.

3.1.7 Affects of inner radius r

Greater values of r would assist the designer in eradicating affects of erosive burning but in order to utilize maximum chamber case available volume it will be desirable to keep r as small as possible. The range for r was set to be between 55 to 79 mm. Fig. 2(e) and Fig. 2(f) shows that by increasing the r , the values of V_l and w_f decrease slowly and F_{av} increases while m_p and t_b decrease. Better neutrality is attainable at higher values of r but compromise is required to be made between better neutrality and m_p while trying to achieve maximum F_{av} . A_{bi} increases by in-

creasing r .

3.1.8 Affects of slot length L_s

Fig. 2(g) and Fig. 2(h) show that F_{av} and m_p increase by increasing the L_s while keeping L_c constant. A_{bi} increases and values of w_f remain constant. Similarly value of t_b decreases because the pressure increases. Better values of neutrality are attainable at higher L_s . The V_1 is not changed much by bringing variations in L_s .

3.1.9 Affects of cylindrical length L_c

Fig. 2(i) and Fig. 2(j) show that F_{av} and m_p increase by increasing the L_c while keeping L_s constant. A_{bi} also increases while values of w_f remain constant. Similarly value of t_b decreases because the pressure increases. No variation is observed in values of neutrality by increasing L_c . So compared to L_s , the L_c has only a little influence on neutrality. The V_1 is not changed much by changing L_c .

3.1.10 Affects of half width of slot x

The range for x was set to be between 5 to 20. From Fig. 2(k) and Fig. 2(l), by increasing value of x , F_{av} , m_p , A_{bi} and V_1 decrease slightly while t_b increases slowly. Neutrality is not affected much by increasing or decreasing x . Values of w_f remains constant by increasing x . Due to small length of slot portion, in the current case, variations in the other parameters caused by changing x are less pronounced. These variations will be much more visible in designs where L_s will be much higher.

Although all the preliminary results fulfill the basic design requirements in terms of average thrust, mass of propellant and burning time. However to come to the optimal result, optimization needs to be conducted.

3.2 Optimization

The optimization toolbox of Matlab FMINCON for local optimization utilizing exact analysis has been used in which FMINCON attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. This is generally referred to as constrain-

ed nonlinear optimization or nonlinear programming. Medium scale optimization has been used in which FMINCON uses a sequential quadratic programming (SQP) method. In this method, the function solves a quadratic programming (QP) subproblem at each iteration. Average thrust has been computed as the objective function with constraints of burning time, mass of propellant, radius and length of grain. Upper and lower limits have been defined for the independent design variables and an initial guess provided.

3.2.1 Objective function

Average thrust F_{av} has been defined as the objective function:

$$\max F_{av}(X)$$

$$X = [N, R, r, L_s, L_c, x]$$

3.2.2 Constraints

The design objectives and other values of parameters become the constraints for the current optimization procedure. Considering the constraints, R has been fixed at 147 mm.

3.2.3 Range of parameter variation

- 1) Half width of slot/mm : $5 \leq x \leq 20$
- 2) Inner radius/mm : $55 \leq r \leq 79$
- 3) Number of slot points : $3 \leq N \leq 8$
- 4) Length of slot/mm : $50 \leq L_s \leq 650$
- 5) Length of cylinder/mm : $3100 \leq L_c \leq 3700$

3.2.4 Initial guess

$$N = 4, x = 5 \text{ mm}, r = 60 \text{ mm}, L_s = 430 \text{ mm}, L_c = 3270 \text{ mm}$$

One handicap of SQP is that it can handle continuous variables only. Hence while conducting optimization, the discrete variable N has been handled by attempting number of optimization cycles over its whole range, one by one, by keeping the values constant. It has been found that the critical constraint of m_p is being violated at values $N \leq 3$ whereas the value of F_{av} is less at $N \geq 5$ compared to the value at $N = 4$. For the current design requirements and constraints, the best solution is only at $N = 4$.

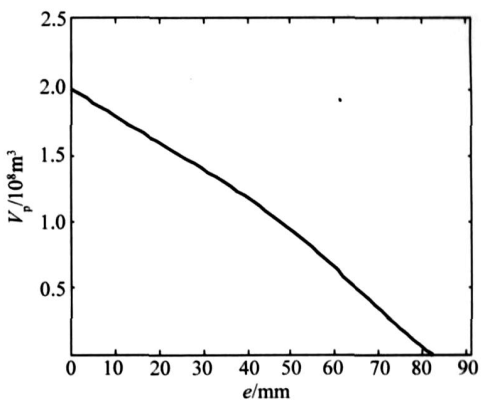
3.2.5 Results

Fig. 3 and Table 2 give optimal results.

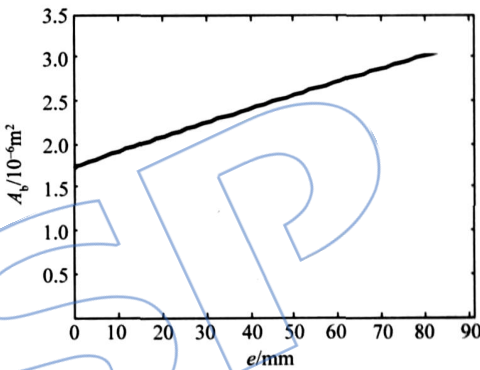
Fig. 4 shows the slotted tube grain.

Table 1 Optimized results of STG

Optimized results	
r / mm	64
x / mm	7.57
L_s / mm	400
L_c / mm	3300
N	4
R / mm	147
F_{av} / kN	245
m_p / kg	360
t_b / s	3.84
n_{eu}	1.39
p_{av} / MPa	33.1
w_f	0.564
V_l	0.84

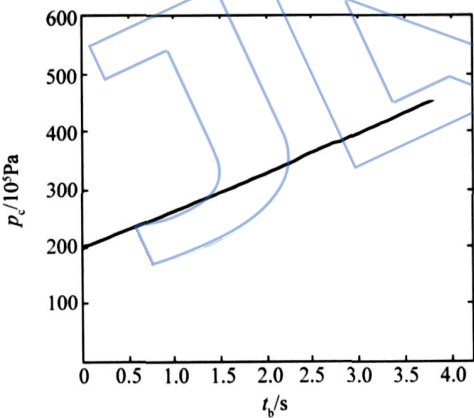


(c) V_p vs e

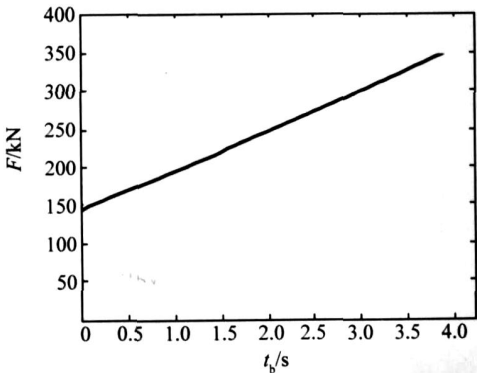


(d) A_b vs e

Fig. 3 Slotted tube grain optimization results



(a) P_c vs t_b



(b) F vs t_b



(a) Inner void



(b) Grain profile

Fig. 4 Slotted tube grain

The results of the optimized SRM STG indicate the following:
With the grain length and diameter fixed,

the mass of propellant required has been successfully acquired. The required mass was (350 ± 10) kg whereas the calculated mass is 360 kg.

The volumetric loading fraction and web fraction are also in accordance with the desired values.

The burning time has been deduced to be 3.84 s whereas the desired limit was (4.5 ± 1) s.

Average thrust required was ≥ 240 kN and it can be seen that the average thrust that has been achieved through the optimized grain is 245 kN.

After optimization, all the design parameters are lying inside the desired ranges. Although all the preliminary results were also qualifying the required design objectives but the value of F_{av} has been further increased and maximized after optimization hence by using this technique of design and optimization an optimal slotted tube configuration has been attained.

4 Conclusion

In the present study, the STG for SRM was to be designed preliminarily to get the maximum possible average thrust while remaining within the constraints of m_p , t_b , L and R . The STG geometry is totally dependent upon N , R , r , L_s , L_c , x . Affects of these variables have been studied. Increase or decrease in the values of these variables bring significant changes in the STG performance. Due to these attributes of STG

configuration, compromises have to be made in selecting values of N , R , r , L_s , L_c , x for designing the grain geometry.

Basing on the initial designs and conclusions derived from the preliminary study, SQP was applied for attaining the optimal design. The results obtained demonstrated that the value of design objective was maximized after optimization while remaining within the desired constraint limits.

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